

Chapter 13

Finance 300
David Moore



Weighted average reminder

Your grade 30% for the midterm 50% for the final. Homework is worth 10% and quizzes another 10%. You did perfect on the homework and quizzes. The midterm you received a 81 and the final was an 92. What is your final grade?

Expected Returns

- Expected returns are based on the probabilities of possible outcomes
- In this context, “expected” means average if the process is repeated many times
- The “expected” return does not even have to be a possible return

$$E(R) = \sum_{i=1}^n p_i R_i$$

Example: Expected Returns

- Suppose you have predicted the following returns for stocks C and T in three possible states of the economy. What are the expected returns?

State	Probability	C	T
Boom	0.3	0.15	0.25
Normal	0.5	0.10	0.20
Recession	???	0.02	0.01

Variance and Standard Deviation

- Variance and standard deviation measure the volatility of returns
- Using unequal probabilities for the entire range of possibilities
- Weighted average of squared deviations

$$\sigma^2 = \sum_{i=1}^n p_i (R_i - E(R))^2$$

Example: Variance and Standard Deviation

- Consider the previous example. What are the variance and standard deviation for each stock?
- Stock C
 - $\sigma^2 = .3(0.15-0.099)^2 + .5(0.10-0.099)^2 + .2(0.02-0.099)^2 = 0.002029$
 - $\sigma = 4.50\%$
- Stock T
 - $\sigma^2 = .3(0.25-0.177)^2 + .5(0.20-0.177)^2 + .2(0.01-0.177)^2 = 0.007441$
 - $\sigma = 8.63\%$

Another Example

- Consider the following information:

<u>State</u>	<u>Probability</u>	<u>ABC, Inc. Return</u>
Boom	.25	0.15
Normal	.50	0.08
Slowdown	.15	0.04
Recession	.10	-0.03

- What is the expected return?
- What is the variance?
- What is the standard deviation?

Portfolios

- A portfolio is a collection of assets
- An asset's risk and return are important in how they affect the risk and return of the portfolio
- The risk-return trade-off for a portfolio is measured by the portfolio expected return and standard deviation, just as with individual assets

Example: Portfolio Weights

- Suppose you have \$15,000 to invest and you have purchased securities in the following amounts. What are your portfolio weights in each security?
 - \$2000 of DIS
 - \$3000 of KO
 - \$4000 of AAPL
 - \$6000 of PG
 - DIS: $2/15 = .133$
 - KO: $3/15 = .2$
 - AAPL: $4/15 = .267$
 - PG: $6/15 = .4$

Portfolio Expected Returns

- The expected return of a portfolio is the weighted average of the expected returns of the respective assets in the portfolio

$$E(R_P) = \sum_{j=1}^m w_j E(R_j)$$

- You can also find the expected return by finding the portfolio return in each possible state and computing the expected value as we did with individual securities

Example: Expected Portfolio Returns

- Consider the portfolio weights computed previously. If the individual stocks have the following expected returns, what is the expected return for the portfolio?
 - DIS: 19.69%
 - KO: 5.25%
 - AAPL: 16.65%
 - PG: 18.24%
- $E(R_p) = .133(19.69\%) + .2(5.25\%) + .267(16.65\%) + .4(18.24\%) = 15.41\%$

Portfolio Variance

- Compute the portfolio return for each state:

$$R_p = w_1R_1 + w_2R_2 + \dots + w_mR_m$$

- Compute the expected portfolio return using the same formula as for an individual asset
- Compute the portfolio variance and standard deviation using the same formulas as for an individual asset

Example: Portfolio Variance

- Consider the following information on returns and probabilities:

- Invest 50% of your money in Asset A

State	Probability	A	B	Portfolio
Boom	.4	30%	-5%	12.5%
Bust	.6	-10%	25%	7.5%

- What are the expected return and standard deviation for each asset?
- What are the expected return and standard deviation for the portfolio?

Systematic Risk

- Risk factors that affect a large number of assets
- Also known as non-diversifiable risk or market risk
- Includes such things as changes in GDP, inflation, interest rates, etc.

Unsystematic Risk

- Risk factors that affect a limited number of assets
- Also known as unique risk and asset-specific risk
- Includes such things as labor strikes, part shortages, etc.

Returns

- Total Return = expected return + unexpected return
- Unexpected return = systematic portion + unsystematic portion
- Therefore, total return can be expressed as follows:
 - Total Return = expected return + systematic portion + unsystematic portion

Diversification

- Portfolio diversification is the investment in several different asset classes or sectors
- Diversification is not just holding a lot of assets
- For example, if you own 50 Internet stocks, you are not diversified
- However, if you own 50 stocks that span 20 different industries, then you are diversified

Table 13.7

TABLE 13.7

Standard Deviations
of Annual Portfolio
Returns

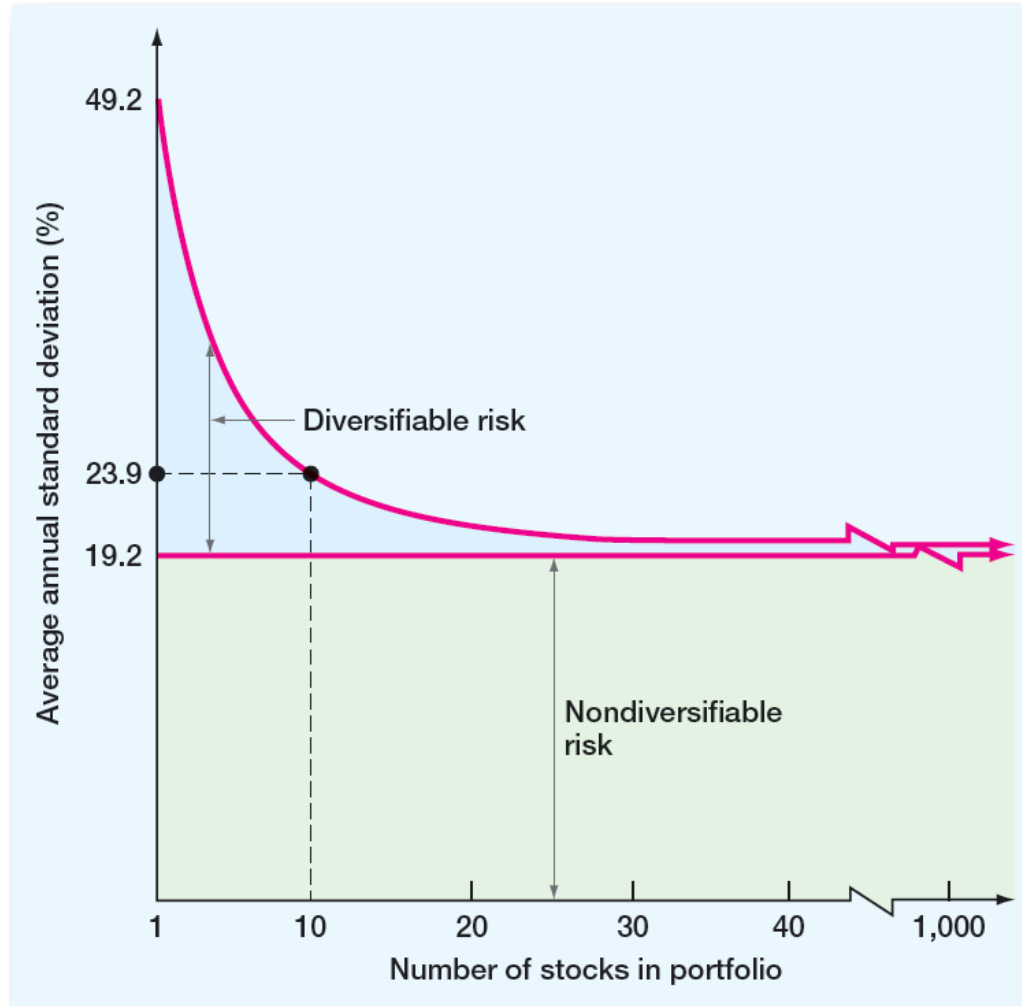
(1) Number of Stocks in Portfolio	(2) Average Standard Deviation of Annual Portfolio Returns	(3) Ratio of Portfolio Standard Deviation to Standard Deviation of a Single Stock
1	49.24%	1.00
2	37.36	.76
4	29.69	.60
6	26.64	.54
8	24.98	.51
10	23.93	.49
20	21.68	.44
30	20.87	.42
40	20.46	.42
50	20.20	.41
100	19.69	.40
200	19.42	.39
300	19.34	.39
400	19.29	.39
500	19.27	.39
1,000	19.21	.39

These figures are from Table 1 in M. Statman, "How Many Stocks Make a Diversified Portfolio?" *Journal of Financial and Quantitative Analysis* 22 (September 1987), pp. 353–63. They were derived from E.J. Elton and M.J. Gruber, "Risk Reduction and Portfolio Size: An Analytical Solution," *Journal of Business* 50 (October 1977), pp. 415–37.

The Principle of Diversification

- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another
- However, there is a minimum level of risk that cannot be diversified away and that is the systematic portion

Figure 13.1



Diversifiable Risk

- The risk that can be eliminated by combining assets into a portfolio
- Often considered the same as unsystematic, unique or asset-specific risk
- If we hold only one asset, or assets in the same industry, then we are exposing ourselves to risk that we could diversify away

Total Risk

- Total risk = systematic risk + unsystematic risk
- The standard deviation of returns is a measure of total risk
- For well-diversified portfolios, unsystematic risk is very small
- Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk

Systematic Risk Principle

- There is a reward for bearing risk
- There is not a reward for bearing risk unnecessarily
- The expected return on a risky asset depends only on that asset's systematic risk since unsystematic risk can be diversified away

Measuring Systematic Risk

- How do we measure systematic risk?
 - We use the beta coefficient
- What does beta tell us?
 - A beta of 1 implies the asset has the same systematic risk as the overall market
 - A beta < 1 implies the asset has less systematic risk than the overall market
 - A beta > 1 implies the asset has more systematic risk than the overall market

Table 13.8 – Selected Betas

	Beta Coefficient (β_i)
Coca-Cola	.35
Kellogg	.57
Microsoft	.69
Google	1.09
3M	1.24
Tesla	1.28
JC Penney	1.61
Abercrombie & Fitch	1.85

Total vs. Systematic Risk

- Consider the following information:

	Standard Deviation	Beta
Security C	20%	1.25
Security K	30%	0.95

- Which security has more total risk?
- Which security has more systematic risk?
- Which security should have the higher expected return?

Example: Portfolio Betas

- Consider the previous example with the following four securities

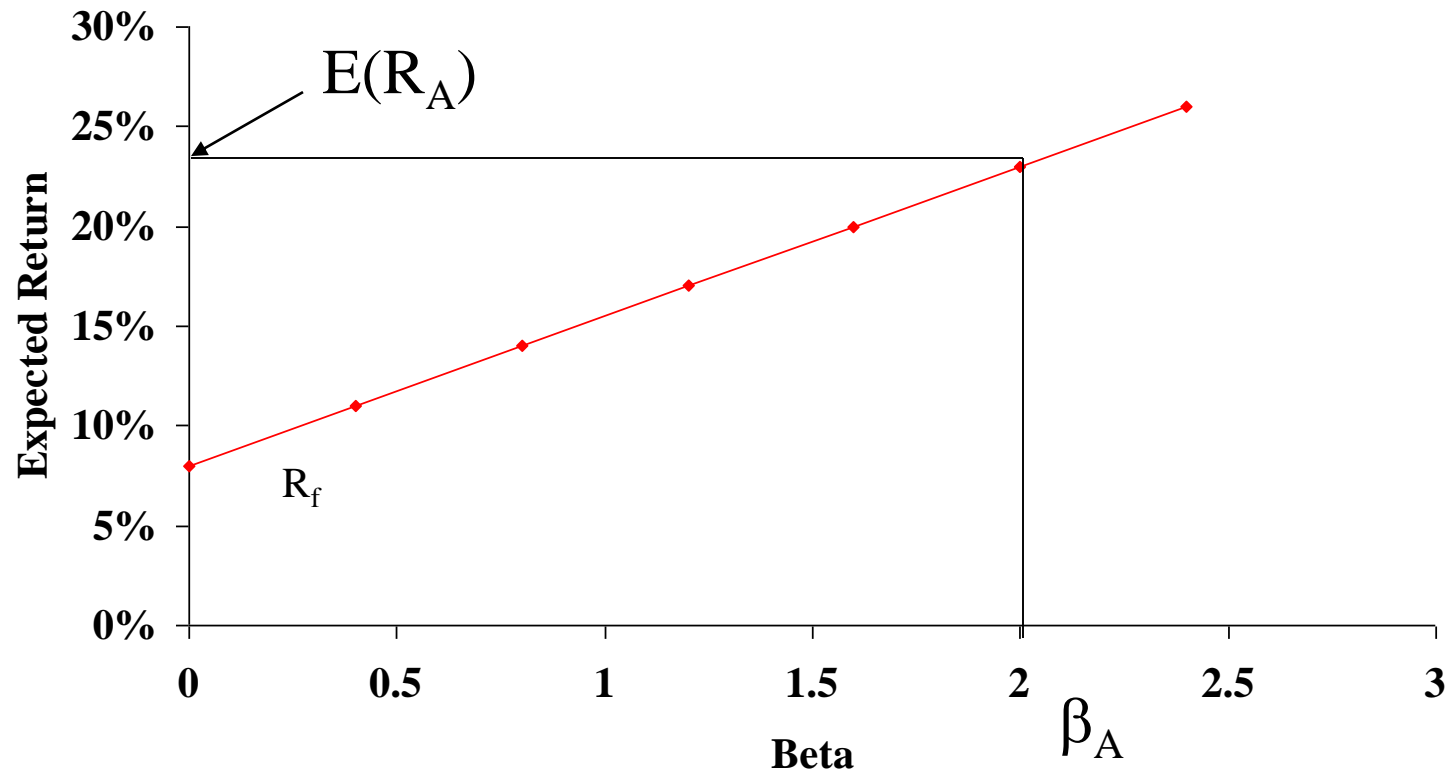
<u>Security</u>	<u>Weight</u>	<u>Beta</u>
DIS	.133	1.444
KO	.2	0.797
AAPL	.267	1.472
PG	.4	0.647

- What is the portfolio beta?
- $.133(1.444) + .2(0.797) + .267(1.472) + .4(0.647) = 1.003$

Beta and the Risk Premium

- Remember that the risk premium = expected return – risk-free rate
- The higher the beta, the greater the risk premium should be
- Can we define the relationship between the risk premium and beta so that we can estimate the expected return?
 - YES!

Example: Portfolio Expected Returns and Betas



Reward-to-Risk Ratio: Definition and Example

- The reward-to-risk ratio is the slope of the line illustrated in the previous example
 - $\text{Slope} = (E(R_A) - R_f) / (\beta_A - 0)$
 - Reward-to-risk ratio for previous example = $(20 - 8) / (1.6 - 0) = 7.5$
- What if an asset has a reward-to-risk ratio of 8 (implying that the asset plots above the line)?
- What if an asset has a reward-to-risk ratio of 7 (implying that the asset plots below the line)?

Market Equilibrium

- In equilibrium, all assets and portfolios must have the same reward-to-risk ratio, and they all must equal the reward-to-risk ratio for the market

$$\frac{E(R_A) - R_f}{\beta_A} = \frac{E(R_M - R_f)}{\beta_M}$$

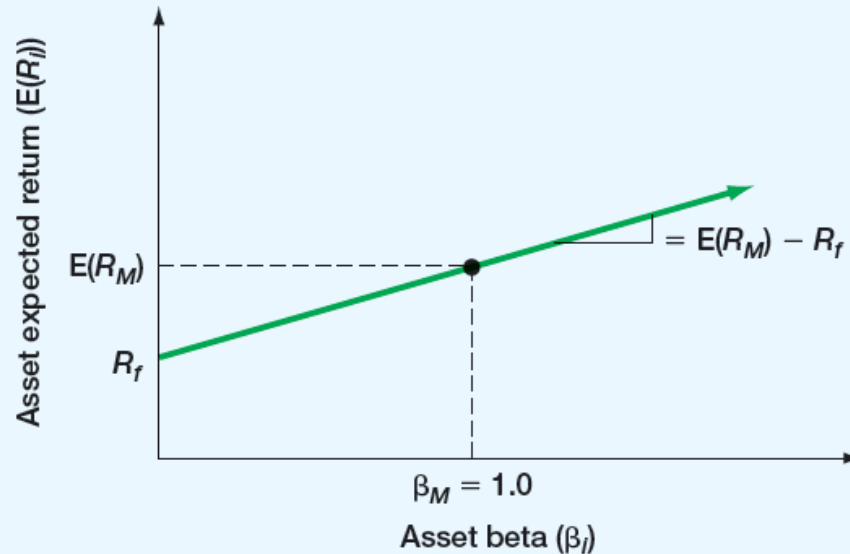
Security Market Line

- The security market line (SML) is the representation of market equilibrium
- The slope of the SML is the reward-to-risk ratio:
 $(E(R_M) - R_f) / \beta_M$
- But since the beta for the market is always equal to one, the slope can be rewritten
- Slope = $E(R_M) - R_f$ = market risk premium

Figure 13.4

FIGURE 13.4

The Security Market Line (SML)



The slope of the security market line is equal to the market risk premium – that is, the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

which is the capital asset pricing model (CAPM).

The Capital Asset Pricing Model (CAPM)

- The capital asset pricing model defines the relationship between risk and return
- $E(R_A) = R_f + \beta_A(E(R_M) - R_f)$
- If we know an asset's systematic risk, we can use the CAPM to determine its expected return
- This is true whether we are talking about financial assets or physical assets

Factors Affecting Expected Return

- Pure time value of money: measured by the risk-free rate
- Reward for bearing systematic risk: measured by the market risk premium
- Amount of systematic risk: measured by beta

Example - CAPM

- Consider the betas for each of the assets given earlier. If the risk-free rate is 4.15% and the market risk premium is 8.5%, what is the expected return for each?

Security	Beta	Expected Return
DIS	1.444	$4.15 + 1.444(8.5) = 16.42\%$
KO	0.797	$4.15 + 0.797(8.5) = 10.92\%$
AAPL	1.472	$4.15 + 1.472(8.5) = 16.66\%$
PG	0.647	$4.15 + 0.647(8.5) = 9.65\%$

Practice 1

- Consider the following information on returns and probabilities:

<u>State</u>	<u>Probability</u>	<u>X</u>	<u>Z</u>
Boom	.25	15%	10%
Normal	.60	10%	9%
Recession	.15	5%	10%

- What are the expected return and standard deviation for a portfolio with an investment of \$6,000 in asset X and \$4,000 in asset Z?

Practice 2

- The risk free rate is 4%, and the required return on the market is 12%.
 - What is the required return on an asset with a beta of 1.5?
 - What is the reward/risk ratio?
 - What is the required return on a portfolio consisting of 40% of the asset above and the rest in an asset with an average amount of systematic risk?